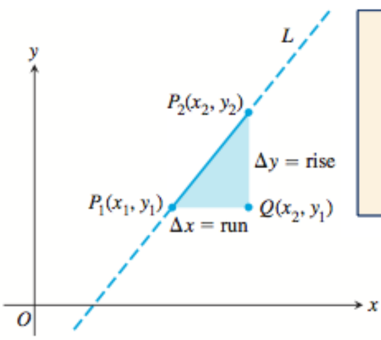
DEFINITION Increments

If a particle moves from the point (x_1, y_1) to the point (x_2, y_2), the **increments** in its coordinates are

$$\Delta x = x_2 - x_1 \quad \text{and} \quad \Delta y = y_2 - y_1.$$



DEFINITION Slope

Let $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ be points on a nonvertical line, L. The **slope** of L is

$$m = \frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}.$$

Parallel Lines

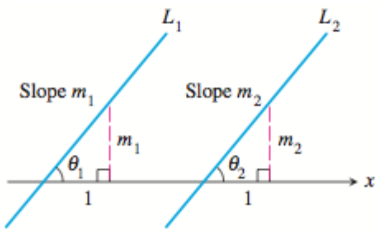


Figure 1.2 If $L_1 \parallel L_2$, then $\theta_1 = \theta_2$ and $m_1 = m_2$. Conversely, if $m_1 = m_2$, then $\theta_1 = \theta_2$ and $L_1 \parallel L_2$.

Perpendicular Lines

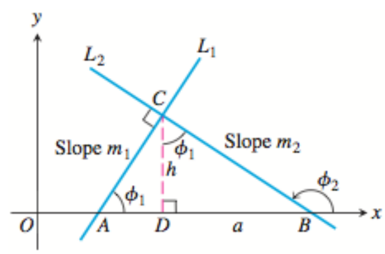


Figure 1.3 $\triangle ADC$ is similar to $\triangle CDB$. Hence ϕ_1 is also the upper angle in $\triangle CDB$, where $\tan \phi_1 = a/h$.

DEFINITION Point-Slope Equation

The equation

$$y = m(x - x_1) + y_1$$

is the **point-slope equation** of the line through the point (x_1, y_1) with slope m.

EXAMPLE 3 Using the Point-Slope Equation

Write the point-slope equation for the line through the point (2, 3) with slope $-3/2$.

$$y = -\frac{3}{2}(x - 2) + 3$$

DEFINITION Slope-Intercept Equation

The equation

$$y = mx + b$$

is the **slope-intercept equation** of the line with slope m and y-intercept b.

EXAMPLE 4 Writing the Slope-Intercept Equation

Write the slope-intercept equation for the line through $(-2, -1)$ and $(3, 4)$.

$$m = \frac{4 - {}^-1}{3 - {}^-2} = \frac{5}{5} = 1$$

$y = (x-3)+4$

$y = x+1$

$y = (x+2)-1$

$y = x+1$

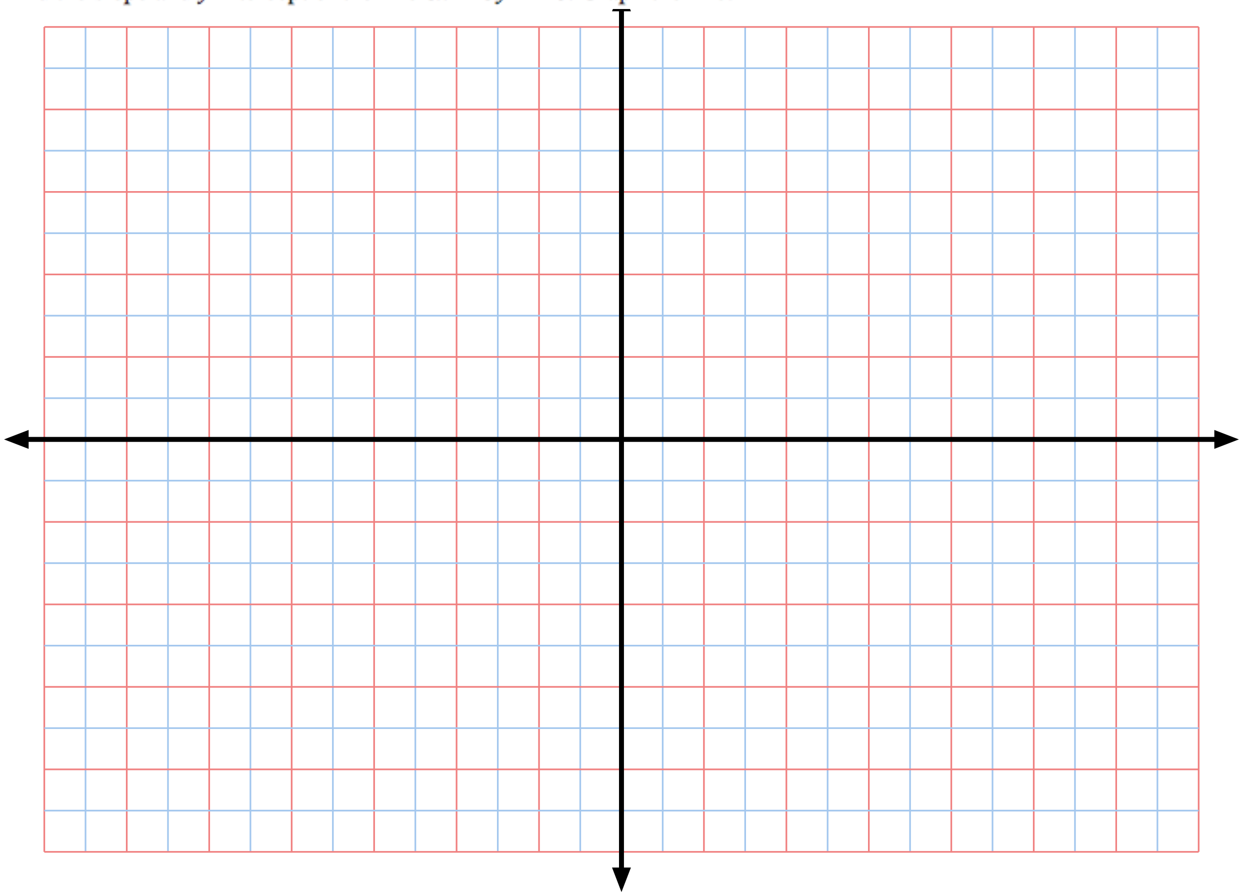
DEFINITION **General Linear Equation**

The equation

$$Ax + By = C \quad (A \text{ and } B \text{ not both } 0)$$

is a **general linear equation** in x and y.

Find the slope and y-intercept of the line $8x + 5y = 20$. Graph the line.



Functions

Euler invented a symbolic way to say "y is a function of x":

$$y = f(x),$$

output

input

EXAMPLE 1 The Circle-Area Function

Write a formula that expresses the area of a circle as a function of its radius. Use the formula to find the area of a circle of radius 2 in.

$$A(r) = \pi r^2$$

$$A(2) = \pi(2)^2 = 4\pi$$

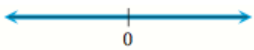
$f(x) = x^2 - 3x + 4$ Find $f(x+2) - f(2)$

$$f(x+2) = (x+2)^2 - 3(x+2) + 4$$
$$= x^2 + 4x + 4 - 3x - 6 + 4$$
$$= x^2 + x + 2$$

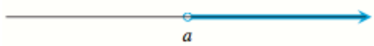
$$f(2) = 2^2 - 3(2) + 4 = 4 - 6 + 4 = 2$$

$$f(x+2) - f(2)$$

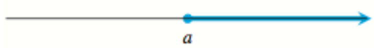
$$x^2 + x + 2 - 2 = x^2 + x$$



Name: The set of all real numbers
Notation: $-\infty < x < \infty$ or $(-\infty, \infty)$



Name: The set of numbers greater than a
Notation: $a < x$ or (a, ∞)



Name: The set of numbers greater than or equal to a
Notation: $a \le x$ or $[a, \infty)$

Name: The set of numbers less than b
Notation: $x < b$ or $(-\infty, b)$

Name: The set of numbers less than or equal to b
Notation: $x \le b$ or $(-\infty, b]$

Figure 1.12 Infinite intervals—rays on the number line and the number line itself. The symbol ∞ (infinity) is used merely for convenience; it does not mean there is a number ∞.

Name: Open interval ab
Notation: $a < x < b$ or (a, b)

Name: Closed interval ab
Notation: $a \le x \le b$ or $[a, b]$

Figure 1.10 Open and closed finite intervals.

Closed at a and open at b
Notation: $a \le x < b$ or $[a, b)$

Open at a and closed at b
Notation: $a < x \le b$ or $(a, b]$

Figure 1.11 Half-open finite intervals.

The endpoints of an interval make up the interval's **boundary** and are called **boundary points.** The remaining points make up the interval's **interior** and are called **interior points. Closed intervals** contain their boundary points. **Open intervals** contain no boundary points. Every point of an open interval is an interior point of the interval.

EXAMPLE 2 Identifying Domain and Range of a Function

Identify the domain and range, and then sketch a graph of the function.

(a) $y = \dfrac{1}{x}$ **(b)** $y = \sqrt{x}$

a) domain: $(-\infty, 0)$ $(0, \infty)$

range: $(-\infty, 0)$ $(0, \infty)$

b domain: $[0, \infty)$

range: $[0, \infty)$

Graph Viewing Skills

1. Recognize that the graph is reasonable.
2. See all the important characteristics of the graph.
3. Interpret those characteristics.
4. Recognize grapher failure.

EXAMPLE 3 Identifying Domain and Range of a Function

Use a grapher to identify the domain and range, and then draw a graph of the function.

(a) $y = \sqrt{4 - x^2}$ **(b)** $y = x^{2/3}$

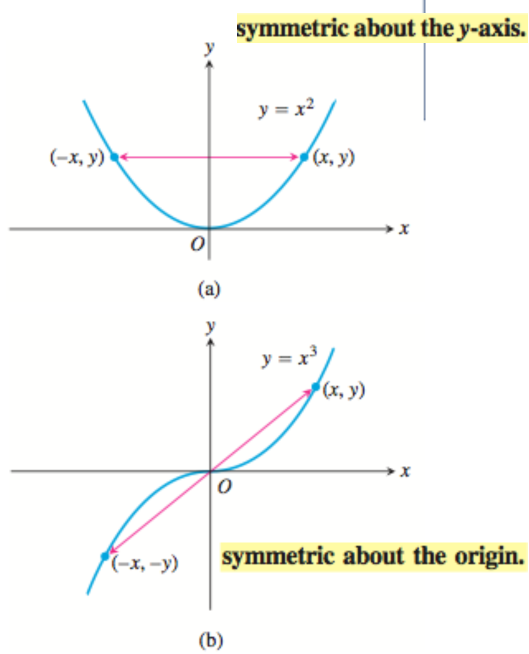
DEFINITIONS Even Function, Odd Function

A function $y = f(x)$ is an

even function of x if $f(-x) = f(x)$,

odd function of x if $f(-x) = -f(x)$,

for every x in the function's domain.



EXAMPLE 4

$f(x) = x^2$

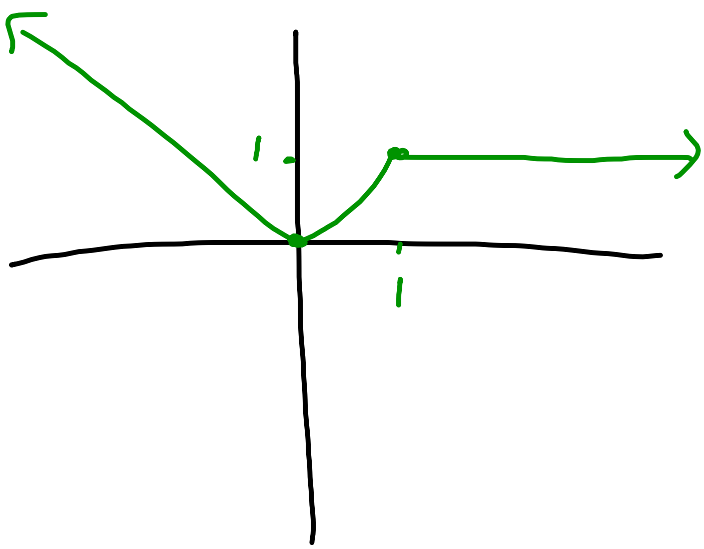
$f(x) = x^2 + 1$

$f(x) = x$

$f(x) = x + 1$

EXAMPLE 5 Graphing Piecewise-Defined Functions

Graph $y = f(x) = \begin{cases} -x, & x < 0 \\ x^2, & 0 \le x \le 1 \\ 1, & x > 1. \end{cases}$



Absolute Value Function

The **absolute value function** $y = |x|$ is defined piecewise by the formula

$$|x| = \begin{cases} -x, & x < 0 \\ x, & x \geq 0. \end{cases}$$

The function is even, and its graph (Figure 1.19) is symmetric about the y-axis.

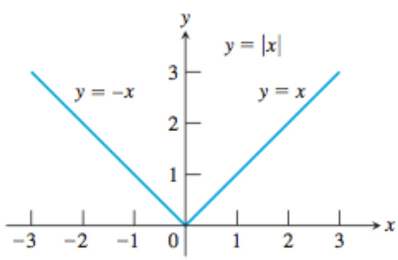


Figure 1.19 The absolute value function has domain $(-\infty, \infty)$ and range $[0, \infty)$.

EXAMPLE 7 Using Transformations

Draw the graph of $f(x) = |x - 2| - 1$. Then find the domain and range.

Composite Functions

EXAMPLE 8 Composing Functions

Find a formula for $f(g(x))$ if $g(x) = x^2$ and $f(x) = x - 7$. Then find $f(g(2))$.

EXPLORATION 1 Composing Functions

Some graphers allow a function such as y_1 to be used as the independent variable of another function. With such a grapher, we can compose functions.

1. Enter the functions $y_1 = f(x) = 4 - x^2$, $y_2 = g(x) = \sqrt{x}$, $y_3 = y_2(y_1(x))$, and $y_4 = y_1(y_2(x))$. Which of y_3 and y_4 corresponds to $f \circ g$? to $g \circ f$?
2. Graph y_1, y_2, and y_3 and make conjectures about the domain and range of y_3.
3. Graph y_1, y_2, and y_4 and make conjectures about the domain and range of y_4.
4. Confirm your conjectures algebraically by finding formulas for y_3 and y_4.

Section 1.3

Grade: «grade»

Subject: «subject»

Date: «date»

DEFINITION Exponential Function

Let a be a positive real number other than 1. The function

$$f(x) = a^x$$

is the **exponential function with base *a*.**

EXAMPLE 1 Graphing an Exponential Function

Graph the function $y = 2(3^x) - 4$. State its domain and range.

EXAMPLE 2 Finding Zeros

Find the zeros of $f(x) = 5 - 2.5^x$ graphically.

Rules for Exponents

If $a > 0$ and $b > 0$, the following hold for all real numbers x and y.

1. $a^x \cdot a^y = a^{x+y}$
2. $\dfrac{a^x}{a^y} = a^{x-y}$
3. $(a^x)^y = (a^y)^x = a^{xy}$
4. $a^x \cdot b^x = (ab)^x$
5. $\left(\dfrac{a}{b}\right)^x = \dfrac{a^x}{b^x}$

Table 1.7 United States Population

Year	Population (millions)	Ratio
1998	276.1	
		$279.3/276.1 \approx 1.0116$
1999	279.3	
		$282.4/279.3 \approx 1.0111$
2000	282.4	
		$285.3/282.4 \approx 1.0102$
2001	285.3	
		$288.2/285.3 \approx 1.0102$
2002	288.2	
		$291.0/288.2 \approx 1.0097$
2003	291.0	

Source: Statistical Abstract of the United States, 2004–2005.

EXAMPLE 3 Predicting United States Population

Use the data in Table 1.7 and an exponential model to predict the population of the United States in the year 2010.

continu

1 Answer (to the nearest tenth in millions)?

DEFINITIONS Exponential Growth, Exponential Decay

The function $y = k \cdot a^x$, $k > 0$ is a model for **exponential growth** if $a > 1$, and a model for **exponential decay** if $0 < a < 1$.

Year	Population (millions)
1880	50.2
1890	63.0
1900	76.2
1910	92.2
1920	106.0
1930	123.2
1940	132.1
1950	151.3
1960	179.3
1970	203.3
1980	226.5
1990	248.7

Source: The Statistical Abstract of the United States, 2004–2005.

Use exponential regression

2 Answer (nearest tenth in millions)?

EXAMPLE 5 Predicting the U.S. Population

Use the population data in Table 1.8 to estimate the population for the year 2000. Compare the result with the actual 2000 population of approximately 281.4 million.

EXAMPLE 6 Interpreting Exponential Regression

What *annual* rate of growth can we infer from the exponential regression equation in Example 5?

3 Answer (%)?